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Retrofit Fault-Tolerant Flight Control Design Under Control Effector Damage

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In this paper a theoretical framework for retrofit reconfigurable flight control is developed and applied to a large class of nonlinear models of aircraft dynamics. The proposed approach solves a difficult problem of retrofit control design that retains a baseline controller and, at the same time, accommodates severe structural damage so that the overall system is stable and the control objective is met. This is accomplished using minimum prior information regarding the baseline controller. It is shown that the retrofit control design strongly depends on the properties of the nominal closed-loop system, which consists of the unperturbed system dynamics and the dynamics of the nominal controller. Using those properties in a judicious manner, it is shown that a retrofit control signal that compensates control effector damage and resulting state-dependent disturbances can be designed using the failure detection and identification subsystem and variable structure adaptation of suitably chosen signals within the proposed retrofit control architecture. In this paper, the focus is on the compensation of the control effector damage and resulting disturbances. It is shown that the proposed retrofit control design results in asymptotic convergence of the tracking error to zero. Its properties are illustrated through simulations of a high-performance aircraft dynamics. The end result of the paper is a retrofit reconfigurable control design framework that is well suited for a large class of problems encountered in reconfigurable flight control.

I. Introduction

A LARGE number of practical technical systems operate over a nominal set of operating regimes that can also be defined as a set in the state space of the system. The system is commonly controlled over the nominal state set by the so-called "trusted" or nominal controller. Such a controller evolved over long periods of time, its dynamics and performance are well understood, and it achieves the objectives over the nominal state set. On the other hand, during the course of operation, it is possible that the system state leaves the nominal state set due to several reasons. For instance, the system may be affected by a disturbance whose magnitude is larger than the largest one for which the nominal controller was tested and tuned. Or different subsystems or component failures may occur which may result in system dynamics that are very different from the nominal one and for which the nominal controller may fail to achieve the control objective. This raises safety issues because controlling the system with the nominal controller outside the nominal state set may result in significant performance deterioration and even instability of the resulting closed-loop system.

An obvious solution to this problem is to replace the nominal controller with an advanced control design that achieves the objective over an enlarged state set. However, this would mean that the effort and funds put into the design of the nominal controller are wasted. In addition, the verification and validation (V&V) of the new controller would need to be performed from the beginning, which is an expensive and often impractical solution.

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An alternative solution is to retain the nominal controller and add a retrofit feedback control module that is active only if the system state leaves the nominal state set. Such an augmented controller would need to be tested only for the operating regimes for which the retrofit module was designed. Moreover, the structure of interconnection between plant and retrofit controller permits the use of existing test and verification tools making the V&V procedure substantially simpler than controller redesign.

The problem of retrofit reconfigurable control design for aerospace applications was considered in [1,2]. The approach is based on a model parameterization and adaptive estimation of parameters of the linearized aircraft model. In that approach, it is assumed that the failures can be represented by a constant vector, and that the aircraft is open loop stable, which may be restrictive assumptions. The proposed scheme estimates all aircraft model parameters (commonly over 100) online and uses these estimates in the control law. Another retrofit control design for adaptive compensation of loss-of-effectiveness failures of the flight control effectors was reported in [3]. Although the approach from that paper is applicable to open-loop unstable plants, the technique was developed for a relatively narrow class of plants and failures. In [4] a control augmentation approach, similar to retrofit control, is proposed where the uncertainty in damage parameters is compensated using the direct adaptive control approach that also compensates for damage-generated nonlinearity using neural networks. The proposed approach assumes full knowledge of the baseline controller structure and dynamics. Augmentation of an existing controller using a neural network element is considered in [5]. It is assumed that only additive uncertainty is present, and its compensation is carried out using online neural networks. The tracking error is shown to be uniformly ultimately bounded.

In this paper a theoretical framework for retrofit reconfigurable flight control is developed and applied to a large class of nonlinear models of aircraft dynamics. The open-loop plant dynamics need not be stable, and the proposed approach extends the results from [3] in that the problem is solved for the case of uncertain plant nonlinearities. Here the results from [6] are extended to a retrofit architecture, and this paper includes a more rigorous proof of stability compared to that from [7]. In addition, the proposed approach compensates for both multiplicative and additive uncertainty. Most importantly, the proposed technique requires some minimum prior knowledge on the structure and dynamics of the

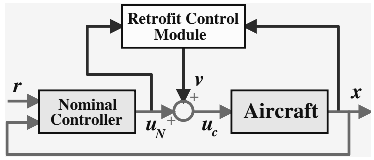

Fig. 1 Structure of the retrofit reconfigurable flight controller.

baseline controller. From that point of view, it can be practically appealing because baseline flight controllers are often designed as several decoupled single-input–single-output (SISO) feedback loops, and compensators are added to minimize the coupling, resulting in a complex architecture for which a corresponding state-space model may be very difficult to derive.

The proposed approach is illustrated in Fig. 1. As seen in the figure, the retrofit reconfigurable flight control module is added to the output of the nominal controller to compensate for the damage.

The proposed approach solves a difficult problem of retrofit control design that retains a baseline controller and, at the same time, accommodates severe structural damage so that the overall system is stable and the control objective is met. This is accomplished using minimum prior information regarding the baseline controller. The design of the algorithms presented in this paper is based on the properties of the nominal closed-loop system, which consists of the unperturbed system dynamics and the dynamics of the nominal controller. The focus is on compensation of the control effector damage and resulting disturbances. The approach is based on adaptive accommodation of uncertainty due to control effector damage and compensation for damage-generated nonlinearity using the variable structure systems approach [8]. It will be shown that the proposed retrofit control design results in asymptotic convergence of the tracking error to zero.

II. Problem Statement

In this paper the focus is on a class of system models that describe both baseline dynamics and dynamics due to damage. The system dynamics is given by

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = f(x) + g(x)Du + \xi(D, x) \tag{2}$$

where $x(t) \in \mathbb{R}^p$ is the state at time t, $x_1 \in \mathbb{R}^{p-n}$, $x_2 \in \mathbb{R}^n$, $x = [x_1^T \quad x_2^T]^T$, $u(t) \in \mathbb{R}^m$ is the control input at time t, D is an unknown diagonal matrix:

$$D = \operatorname{diag}[d_1, d_2, \ldots, d_m]$$

and ξ is an unknown nonlinearity. The diagonal entries of D represent the amount of control effector damage and ξ represents the changes in dynamics resulting from damage. One can scale the diagonal entries of D to be within the interval [0, 1] so that $d_i = 1$ means no damage to the ith control effector, $0 < d_i < 1$ means partial damage, and $d_i = 0$ means total damage. Accordingly, $D = I$ when there is no damage. Since ξ is the change in dynamics resulting from damage,

$$\xi(I, x) = 0 \quad \text{for all } x$$

when there is no damage. The plant dynamics (1) and (2) with no damage will be called the *nominal plant*.

The following assumption will be made throughout the paper:

Assumption 1: The plant (1) and (2) satisfies the following:

1) $f(\cdot)$ and $g(\cdot)$ are smooth functions.

2) There exists a real number $\delta > 0$ such that $g(x)g(x)^T \geq \epsilon I$ for all $x \in \mathbb{R}^p$.

3) $\|g(x)\| \leq c_g$ for all $x \in \mathbb{R}^p$.

4) The set of admissible control effector damage matrices is given by

$$\mathcal{D} = \{D = \operatorname{diag}[d_1, \ldots, d_m] : \epsilon_i \leq d_i \leq 1 \text{ for all } i\} \tag{3}$$

where $\epsilon_i = 0$ for up to $m - n$ effectors, and $\epsilon_i > 0$ are small numbers for the remaining effectors.

5) The unknown nonlinearity ξ resulting from damage satisfies

$$\|\xi(D, x)\| \leq c_0(x) \quad \text{for all } x \in \mathbb{R}^p; \quad \text{and} \quad D \in \mathcal{D}$$

for some known continuous positive semidefinite function c_0.

6) All the states of the plant are available noise free for feedback.

Typical aircraft states that describe dominant dynamics are $(\phi, \theta, \psi, p, q, r, V, \alpha, \beta, h)$. The class of nonlinear plants (1) and (2) is seen to be in the controllable canonical form. This form is obtained when, for instance, a model of aircraft dynamics is transformed to the space of Euler angles (ϕ, θ, ψ) and their rates $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ instead of the original state space $(\phi, \theta, \psi, p, q, r)$, while the states α, β, V, and h remain unchanged. More details regarding such a transformation can be found in [6].

The smoothness assumption on $f(\cdot)$ and $g(\cdot)$ can be relaxed to those that guarantee existence and uniqueness of solutions. In addition, $f(\cdot)$ and $g(\cdot)$ are required to be bounded in every nonempty compact subset of $\mathbb{R}^p$. Note that gg^T is always positive semidefinite. The assumption on gg^T implies that gg^T is invertible and that the inverse is bounded. In fact, it can be shown that

$$[g(x)g(x)^T]^{-1} \leq \frac{1}{\epsilon} I$$

for all $x \in \mathbb{R}^p$. It is easy to show that the assumption is equivalent to

$$\operatorname{rank} g(x) = n \quad \text{for all } x \in \mathbb{R}^p \quad \text{and} \quad m \geq n$$

that is, the rank of $g(x)$ is equal to the number of states for all x and the number of control inputs m is greater than or equal to the number of states. Furthermore, because the range of $g(x)$ is equal to n, the uncertainty is matched. This assumption is used to define the adaptive control law given later in the paper. The condition in the definition of $\mathcal{D}$ ensures that there are fully or partially functional control effectors to achieve the control objectives even after damage.

It is seen that it is assumed that up to $n - m$ effectors can be fully damaged, while the remaining effectors can be partially damaged. This is a reasonable assumption because only a limited amount of damage is allowed (e.g., damage of some of the effectors on one wing) which results in some effectors being fully damaged, while the remaining ones are either partially damaged or fully operational.

The assumption on the size of the change in dynamics due to damage is reasonable. The upper bound for ξ will in general depend on both D and x. Because $\mathcal{D}$ is a compact set, one can take the maximum of the upper bound over $\mathcal{D}$ to get to the upper bound c_0 in the assumption.

An important issue is the choice of the nonlinear function $c_0(x)$. A function that tightly bounds the effect of control effector damage can lead to better performance, whereas a conservative upper bound can lead to control saturation, as the control law given later in the paper depends on c_0, and eventual instability. At present, very little is known about the aerodynamic effects of damage and reasonable estimates of the structure and parameters of c_0 are not available. We expect the situation to change with the renewed interest in aviation safety. For a numerical example given later, we assume a model structure and model parameter values to illustrate the control design.

The retrofit reconfigurable control design is based on an assumption that a controller that achieves desired closed-loop stability and performance for the nominal plant is given. This controller will be called the *nominal controller* and the corresponding closed-loop system will be called the *nominal closed-loop system*. The objective of retrofit controller design is to design an adaptive control law for the plant (1) and (2) that guarantees, in the presence of control effector damage, similar closed-loop stability and performance, if possible, as the nominal controller for the nominal plant. To state this problem and the underlying assumptions precisely, the nominal controller and the nominal closed-loop system properties are studied first.

Nominal Controller and Closed-Loop System: The nominal controller is given by

$$\dot{\omega} = h(\omega, x, r) \tag{4}$$

$$u_N = q(\omega, x, r) \tag{5}$$

where $\omega(t) \in \mathbb{R}^l$ is the controller state at time t, $x(t)$ is the plant state at time t, and r is a reference input. The nominal closed-loop system is easily seen to be

$$\dot{x} = f(x) + g(x)q(\omega, x, r) \tag{6}$$

$$\dot{\omega} = h(\omega, x, r) \tag{7}$$

after setting $D = I, \xi(I, \cdot) = 0$ for the nominal plant and $u = u_{\text{nom}}$.

Nominal Closed-Loop System Properties: Let $\gamma > 0$ be a given number and

$$\mathcal{R} = \{r\colon [0,\infty) \to \mathbb{R}^k | r \text{ is piecewise continuous and } \sup_{t\in[0,\infty)} \|r(t)\| < \gamma\} \tag{8}$$

be a set of reference inputs. This large set of reference inputs contains all the typical reference/command inputs used in flight control.

For each reference input $r \in \mathcal{R}$, let

$$\phi(t; x(0), \omega(0), r) = \begin{bmatrix} \phi_x(t; x(0), \omega(0), r) \\ \phi_\omega(t; x(0), \omega(0), r) \end{bmatrix} \tag{9}$$

denote the solution of the nominal closed-loop system equations (6) and (7) at time t starting at the initial state $(x(0), \omega(0))$. That is,

$$\dot{\phi}_x = f(\phi_x) + g(\phi_x)q(\phi_\omega, \phi_x) \tag{10}$$

$$\dot{\phi}_\omega = h(\phi_\omega, \phi_x, r) \tag{11}$$

$$\phi_x(0; x(0), \omega(0), r) = x(0) \quad \text{and} \quad \phi_\omega(0; x(0), \omega(0), r) = \omega(0) \tag{12}$$

where the equations in (12) means that ϕ satisfies the specified initial conditions. The objective of nominal controller design is to guarantee certain stability and tracking properties for the trajectories $\phi(t; x(0), \omega(0), r)$. These properties may be as follows: for each $r \in \mathcal{R}$

1) The trajectory $\phi(t; 0, 0, r)$ starting at the initial state (0, 0) is the desired closed-loop system behavior in response to the reference input r, and

2) The neighboring trajectories $\phi(t; x(0), \omega(0), r)$ starting from initial states $(x(0), \omega(0))$ close to (0, 0) converge to $\phi(t; 0, 0, r)$ asymptotically.

At the flight control design stage, the requirements on the closed-loop response are given in terms of low-order equivalent systems (LOES) for certain flight variables or in terms of well-known frequency domain flying qualities criteria. Such requirements are used to design and verify the nominal controller (4) and (5). Therefore, in the case of a successful control design, the nominal closed-loop response $\phi(t; 0, 0, r)$ is close in some appropriate sense to the response corresponding to the requirements. Henceforth, the achieved nominal response $\phi(t; 0, 0, r)$ will be refered to as the *desired* system behavior.

The state (0, 0) is used as a "reference point" to specify the desired system behavior in the above discussion. This is purely for convenience. For each $r \in \mathcal{R}$, the desired system behavior may be given as

$$\phi(t; \bar{x}_r, \bar{\omega}_r, r)$$

that is, the solution starting from some specified reference point that may depend on r. One can perform a change of variables to move the reference point to (0, 0).

Assumption 2: For each reference input $r \in \mathcal{R}$, the desired system behavior $\phi(t; 0; 0, r)$ has the following properties:

1) $\phi(t; 0; 0, r)$ is a bounded function of time.

2) For each initial state $(x(0), \omega(0))$, $\phi(t; x(0), \omega(0), r)$ is bounded and

$$\lim_{t\to\infty} [\phi_x(t; 0, 0, r) - \phi_x(t; x(0), \omega(0), r)] = 0$$

The first statement says that the nominal controller guarantees that the closed-loop system solution starting at the reference point is bounded. This assumption is reasonable in practice. The second statement says that the nominal system state is bounded, and the tracking error converges to zero.

Additional information regarding stability and tracking of the nominal system may be available in some cases. To describe such information, define the nominal closed-loop tracking error:

$$\begin{bmatrix} e_x(t; x(0), \omega(0), r) \\ e_\omega(t; x(0), \omega(0), r) \end{bmatrix} = \phi(t; 0, 0, r) - \phi(t; x(0), \omega(0), r) \quad \text{for all } t \geq 0 \tag{13}$$

for each initial state $(x(0), \omega(0))$ and each reference input $r \in \mathcal{R}$. Note that the tracking error consists of errors in nominal plant states (e_x) and nominal controller states (e_ω). The tracking error satisfies a differential equation of the form:

$$\begin{bmatrix} \dot{e}_x(t; x(0), \omega(0), r) \\ \dot{e}_\omega(t; x(0), \omega(0), r) \end{bmatrix} = \dot{\phi}(t; 0, 0, r) - \dot{\phi}(t; x(0), \omega(0), r) = \tilde{f}(t; e_x, e_\omega, \phi^*, r) \tag{14}$$

where $\phi^*(t) = \phi(t; 0, 0, r) = [x^{*T}(t), \omega^{*T}(t)]$ and

$$\tilde{f} = \begin{bmatrix} f(x) - f(x^*) + g(x)q(\omega, x, r) - g(x^*)q(\omega^*, x^*, r) \\ h(\omega, x, r) - h(\omega^*, x^*, r) \end{bmatrix} \tag{15}$$

Moreover, for zero initial conditions, one has

$$e_x(t; 0, 0, r) = 0 \quad \text{and} \quad e_\omega(t; 0, 0, r) = 0 \quad \text{for all } t \geq 0 \text{ and for all } r \in \mathcal{R}$$

by definition.

Assumption 3:

1) Let ζ denote a bounded disturbance such that $\eta \in \mathcal{S}_\zeta = \{\zeta\colon \|\zeta\| \leq c_\zeta\}$. The system

$$\begin{bmatrix} \dot{e}_x(t; x(0), \omega(0), r) \\ \dot{e}_\omega(t; x(0), \omega(0), r) \end{bmatrix} = \tilde{f}(t; e_x, e_\omega, \phi_o, r) + \zeta \tag{16}$$

has the following properties: i) It is bounded-input–bounded-output (BIBO) stable; and ii) $\lim_{t\to\infty} \zeta(t) = 0$ implies that $\lim_{t\to\infty} e_x(t) = 0$ and $\lim_{t\to\infty} e_\omega(t) = 0$.

2) Functions $h(\omega, x, r)$ and $q(\omega, x, r)$ from the definition of the nominal controller satisfy

$$h(\omega, x, r,) = h_0(\omega, x_1, r) + H(\omega, x_1, r)x_2 \tag{17}$$

$$q(\omega, x, r,) = q_0(\omega, x_1, r) + Q(\omega, x_1, r)x_2 \tag{18}$$

where $H(\omega, x_1, r)$ and $Q(\omega, x_1, r)$ are bounded for all values of the arguments.

The first assumption is reasonable because the nominal closed-loop system is commonly designed to be robust to external disturbances and achieves asymptotic tracking when the disturbance is removed. The second assumption states that the controller is linear in x_2, which is the only prior information (besides the signal u_N) regarding the control law that is needed to implement the proposed retrofit controller.

Retrofit Control Problem Statement: The retrofit control problem can be stated as follows. Consider the plant (1) and (2) with damage modeled by D and ξ, and the nominal controller (4) and (5). The retrofit control design problem is to find a feedback control law v such that the total control input

$$u = u_N + v \tag{19}$$

achieves the desired closed-loop system response in the presence of damage. For instance, for each reference input $r \in \mathcal{R}$, there is a neighborhood of zero such that the tracking errors starting from any initial condition in the neighborhood tends to zero asymptotically for any damage $D \in \mathcal{D}$.

III. Retrofit Controller Design for Control Effector Damage Accommodation

The retrofit controller design consists of the design of an observer for estimating damage and the design of a retrofit signal v. The equations describing the controller will be derived for the case when the nominal closed-loop system satisfies Assumptions 2 and 3. Details of the design stages for the nonlinear plant (1) and (2) are given first. This is followed by the properties guaranteed by the designs. Then, the effects of control constraints are discussed. Finally, the design procedure is illustrated through simulations.

A. Observer Design

Since D is a diagonal matrix, one can write

$$Du = Ud$$

where

$$U = \text{diag}[u_1, u_2, \ldots, u_m], \qquad d = [d_1, d_2, \ldots, d_m]^T$$

and u_i is the ith entry of u. The plant equation (2) can now be written as

$$\dot{x}_2 = f(x) + g(x)Ud + \xi(d, x) \tag{20}$$

where d and ξ represent damage and are unknown.

Let $\Lambda \in \mathbb{R}^{n\times n}$ and $\Gamma \in \mathbb{R}^{m\times m}$ be positive-definite matrices. Define the following observer:

$$\dot{\hat{x}}_2 = f(x) + g(x)U\hat{d} + \hat{\xi} - \Lambda(\hat{x}_2 - x_2) \tag{21}$$

$$\dot{\hat{d}} = \text{Proj}_{[\epsilon,1]}\{-\Gamma U g^T(x)(\hat{x}_2 - x_2)\} \tag{22}$$

$$\hat{\xi} = -\text{sgn}((\hat{x}_2 - x_2))c_0(x) \tag{23}$$

where the projection operator $\text{Proj}_{[\epsilon,1]}\{\cdot\}$ is used to keep the estimate $\hat{d}_i$ of damage to the ith control effector within the interval $[\epsilon_i, 1]$, and the sign function sgn is given by

$$\text{sgn}\,(z) = \begin{cases} z/\|z\| & \text{if } \|z\| > 0 \\ 0 & \text{otherwise} \end{cases}$$

For more details on the projection operator, see the Appendix.

The state and damage estimates are initialized as

$$\hat{x}_2(0) = x_2(0) \quad \text{and} \quad \hat{D}(0) = I \tag{24}$$

As a consequence, in the no-damage case it follows that $\hat{x}_2(t) = x_2(t)$ for all time.

Now, define the state estimation error:

$$\hat{e}_x = \hat{x}_2 - x_2 \tag{25}$$

and the damage parameter estimation error:

$$\hat{e}_d = \hat{d} - d \tag{26}$$

Using the fact that d is a constant, we obtain

$$\dot{\hat{e}}_x = -\Lambda\hat{e}_x + g(x)U\hat{e}_d + (\hat{\xi} - \xi(d, x)) \tag{27}$$

$$\dot{\hat{e}}_d = \text{Proj}_{[\epsilon,1]}\{-\Gamma U g^T(x)\hat{e}_x\} \tag{28}$$

$$\hat{\xi} = -\text{sgn}(\hat{e}_x)c_0(x) \tag{29}$$

as the equations of estimation error dynamics. Note that the error dynamics depends on plant state x, damage model parameter d, control input u, and tracking errors.

Theorem III.1: Adaptive algorithms (28) and (29) assure that $\hat{e}_x \in \mathcal{L}^\infty \cap \mathcal{L}^2$.

The proof of the Theorem is given in Appendix A.

Some remarks on the design variables Λ and Γ are in order. Their selection is commonly based on a tradeoff among several conflicting factors (open-loop aircraft dynamics, available persistent excitation in a specific operating regime, desired speed of response of the adaptive system, etc.). A rule of thumb is to choose Λ so that the largest eigenvalue of $-\Lambda$ gives a decay rate that is 2 or 3 times faster than the desired control system response. A large value of Γ results in fast convergence of parameter estimates (not necessarily to their true values). However, if these values are too large, the response may become oscillatory. On the other hand, too small values of Γ result in slow convergence so an optimum value is commonly found through simulations.

B. Retrofit Control Signal Design

We assume that the nominal closed-loop system given by Eqs. (6) and (7) satisfies design requirements, and the only additional information is given in Assumption 3.2 and by the fact that $u_N = q(\omega, x, r)$ is available.

To design the retrofit control signal, we rewrite the observer Eq. (21) as

$$\begin{aligned}\dot{\hat{x}}_2 &= f(x_1, x_2) + g(x_1, x_2)\hat{D}(u_N + v) + \hat{\xi} - \Lambda(\hat{x}_2 - x_2) \\ &= f(x_1, x_2) + g(x_1, x_2)u_N + g(x_1, x_2)(\hat{D} - I)u_N \\ &\quad + g(x)\hat{D}v + \hat{\xi} - \Lambda\hat{e}_x\end{aligned}$$

The retrofit control signal is chosen as

$$v = v_1 + v_2 \tag{30}$$

where v_1 is a solution to the equation:

$$g(x_1, x_2)(\hat{D} - I)u_N + g(x)\hat{D}v_1 + \hat{\xi} = 0 \tag{31}$$

and v_2 is to be determined.

The objective now is to express the plant nonlinearities f and g in terms of $\hat{x}_2$ rather than x_2 so that the properties of the nominal system can be used. Using Assumption 3.2 we further have

$$\begin{aligned}\dot{\hat{x}}_2 &= f(x_1, \hat{x}_2) + f(x_1, x_2) - f(x_1, \hat{x}_2) + g(x_1, \hat{x}_2)(q(\omega, x_1, \hat{x}_2, r) \\ &\quad - Q(\omega, x_1, r)\hat{e}_x) + (g(x_1, x_2) - g(x_1, \hat{x}_2))u_N + g(x)\hat{D}v_2 - \Lambda\hat{e}_x\end{aligned}$$

The signal v_2 is now chosen as a solution of the equation:

$$f(x_1, x_2) - f(x_1, \hat{x}_2) + g(x_1, x_2)u_N - g(x_1, \hat{x}_2)u_N + g(x)\hat{D}v_2 = 0 \tag{32}$$

resulting in

$$\begin{aligned}\dot{\hat{x}}_2 &= f(x_1, \hat{x}_2) + g(x_1, \hat{x}_2)q(\omega, x_1, \hat{x}_2, r) - (g(x_1, \hat{x}_2, r)Q(\omega, x, r) \\ &\quad + \Lambda)\hat{e}_x\end{aligned} \tag{33}$$

The total retrofit signal $v = v_1 + v_2$ is now

$$v = \tilde{g}^{+}(x)[f(x_1, \hat{x}_2) - f(x_1, x_2) + (g(x_1, \hat{x}_2) - g(x_1, x_2))u_N + g(x)(I - \hat{D})u_N - \hat{\xi}] \tag{34}$$

where $\tilde{g}(x)^{+}$ is the Moore–Penrose inverse of $g(x)\hat{D}$.

The proposed retrofit control law is of the augmentation type since it, in general, does not cancel the baseline control signal. To show that, let $\hat{e}_x = 0$, that is, $x_2 = \hat{x}_2$. If, in addition, ξ is small, one has that $v = (I - \hat{D})u_N$. Hence, when $\hat{D}$ is small, the contribution of v approaches u_N. When $\hat{D}$ is close to identity, the contribution of v is close to zero.

C. Closed-Loop System and Its Properties

The following coordinate transformation is considered next:

$$z_1 = x_1, \qquad z_2 = \hat{x}_2, \qquad z_3 = \omega \tag{35}$$

which, using Eq. (33) and Assumption 3.2, results in

$$\dot{z}_1 = z_2 - \hat{e}_x \tag{36}$$

$$\dot{z}_2 = f(z) + g(z)q(z, r) - (g(z, r)Q(z_1, z_3, r) + \Lambda)\hat{e}_x \tag{37}$$

$$\dot{z}_3 = h(z, r) + H(z_1, z_3, r)\hat{e}_x \tag{38}$$

Subtracting the desired dynamics defined at Eq. (15), one obtains

$$\dot{\tilde{z}} = \tilde{f}(z, \phi^*, r) + R(z, r)\hat{e}_x \tag{39}$$

where $\tilde{z}_1 = x_1 - \phi^*_{x1}$, $\tilde{z}_2 = \hat{x}_1 - \phi^*_{x2}$, $\tilde{z}_3 = \omega - \phi^*_{\omega}$, and $R(z, r) = [-I - (g(z, r)Q(z_1, z_3, r) + \Lambda)^T H^T(z_1, z_3, r)]^T$.

The following theorem is considered next.

Theorem III.2: Suppose that the plant (1) and (2) satisfies Assumption 1 and the nominal closed-loop system satisfies Assumptions 2 and 3. Let $r \in \mathcal{R}$ be a reference input and $\phi(t; 0, 0, r) = \phi^*(t)$ be the corresponding desired response. For any initial condition

$$\begin{bmatrix} x(0)^T & \omega(0)^T & \hat{e}_x(0)^T & \hat{e}_d(0)^T \end{bmatrix}^T$$

and for any damage $D \in \mathcal{D}$ and $\|\xi(x)\| \leq c_0(x)$, the states of the closed-loop system (27–34) are bounded and the closed-loop plant tracking error $e = x - \phi^*$ tends to zero as t tends to infinity.

A proof of the result is given in Appendix A.

D. Retrofit Control Design in the Presence of Input Constraints

In practical applications the designer is faced with control input constraints that need to be taken into account to be able to certify the flight control software. An approach that takes into account such constraints is further proposed in the context of retrofit adaptive reconfigurable control design described in this paper.

It is assumed that $u \in \mathcal{S}_u = \{u\colon (u_i)_{\min} \leq u_i \leq (u_i)_{\max}, i = 1, 2, \ldots, m\}$. The retrofit control signal (34) is first rewritten in the form:

$$g(x)v = \eta \tag{40}$$

where

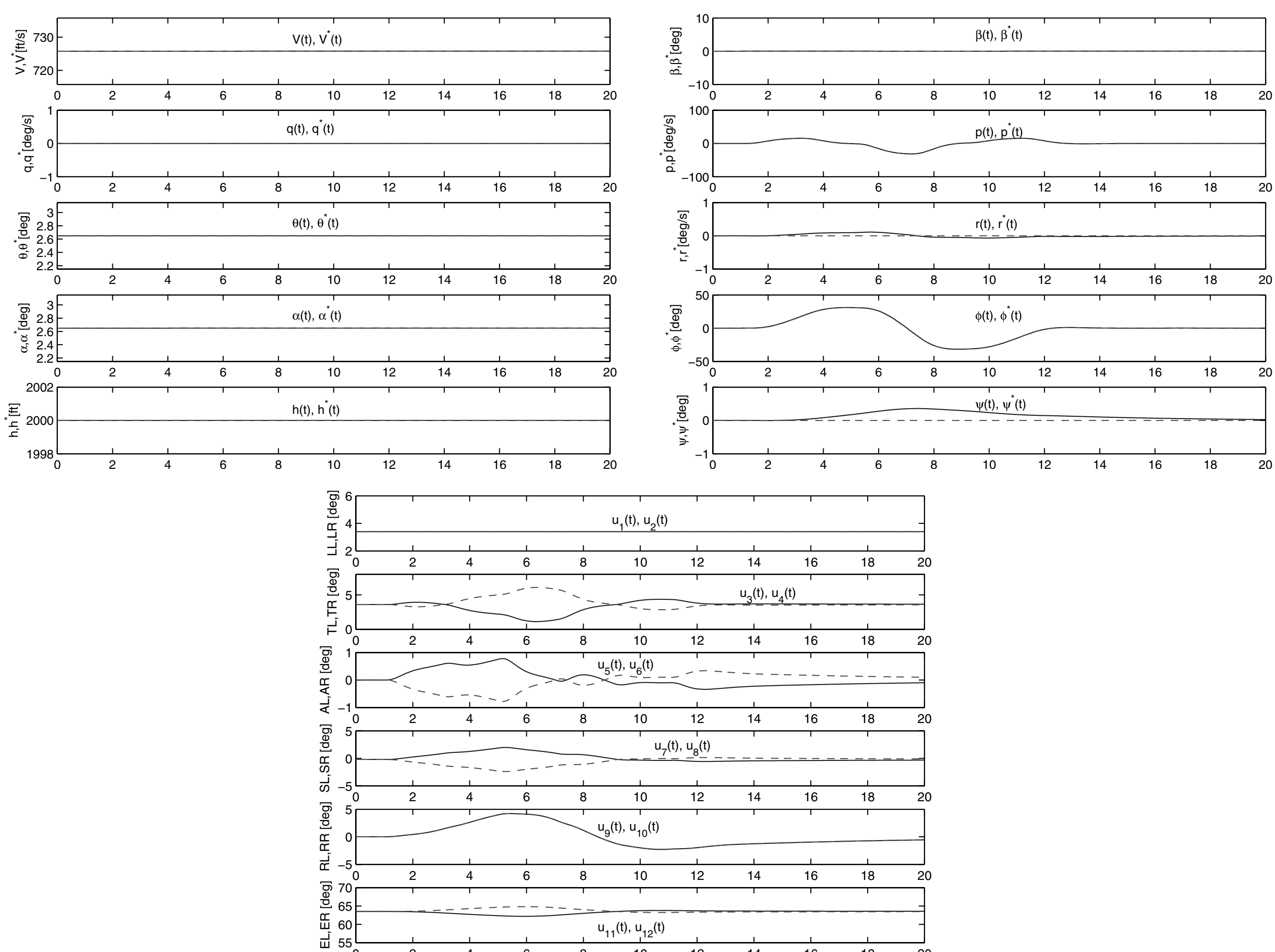

Fig. 2 Response with the baseline controller in the no-damage case.

$$\eta = f(x_1, \hat{x}_2) - f(x_1, x_2) + (g(x_1, \hat{x}_2) - g(x_1, x_2))u_N + g(x)(I - \hat{D})u_N - \hat{\xi}$$

Now the following optimization problem is considered: *minimize:*

$$(g(x)v - \eta)^T Q(g(x)v - \eta) + v^T R v \tag{41}$$

subject to $u = u_N + v \in \mathcal{S}_u$.

In the above statement, $Q = Q^T > 0$ and $R = R^T \geq 0$.

Rate limits can be taken into account through $\dot{u} = \dot{u}_N + \dot{v}$ by approximating the derivatives through discretization and expressing the constraint on $v(t)$ in terms of the current and previous values of $u(t)$ and $u_N(t)$.

IV. Simulations

A simplified version of the algorithms proposed in this paper was recently tested through piloted simulations of a F/A-18 aircraft resulting in excellent response [9]. In this section we show the simulation results obtained using the algorithms presented in this paper. The proposed approach is tested on a medium-fidelity simulation of F/A-18 dynamics in an up-and-away flight regime. The simulation consists of linear stability and control derivatives, nonlinear kinematics and gravity effects, second-order actuator dynamics, and position and rate limits on the flight control effectors.

The states of the model are as follows: $\bar{x}_1$—total velocity (ft/s); $\bar{x}_2$—pitch rate (deg/s); $\bar{x}_3$—pitch angle (deg); $\bar{x}_4$—angle of attack (deg); $\bar{x}_5$—altitude (ft); $\bar{x}_6$—side-slip angle (deg); $\bar{x}_7$—roll rate (deg/s); $\bar{x}_8$—yaw rate (deg/s); $\bar{x}_9$—roll angle (deg); and $\bar{x}_{10}$—yaw angle (deg). To put the model in the controllable canonical form, a simple transformation is introduced:

$$x_1 = \bar{x}_3$$

$$x_2 = \bar{x}_5$$

$$x_3 = \bar{x}_9$$

$$x_4 = \bar{x}_{10}$$

$$x_5 = \bar{x}_2$$

$$x_6 = V_T(\bar{x}_3 - \bar{x}_4)$$

$$x_7 = \bar{x}_7$$

$$x_8 = \bar{x}_8$$

$$x_9 = \bar{x}_1$$

$$x_{10} = \bar{x}_6$$

where V_T denotes the trim velocity. Hence x_6 is a scaled flight-path angle η.

Control inputs are u_1, u_2—leading-edge flaps (left—LL, right—LR); u_3, u_4—trailing-edge flaps (left—TL, right—TR); u_6, u_6—ailerons (left—AL, right—AR); u_7, u_9—stabilators (left—SL, right—SR); u_9, u_{10}—rudders (left—RL, right—RR); and u_{11}, u_{12}—engines (left—EL, right—ER). The positions of $u_1, u_2, \ldots, u_{10}$ are given in degrees, while the engines are characterized by their power level angle (PLA) positions, also given in degrees.

The test maneuver is a 30 deg lateral doublet.

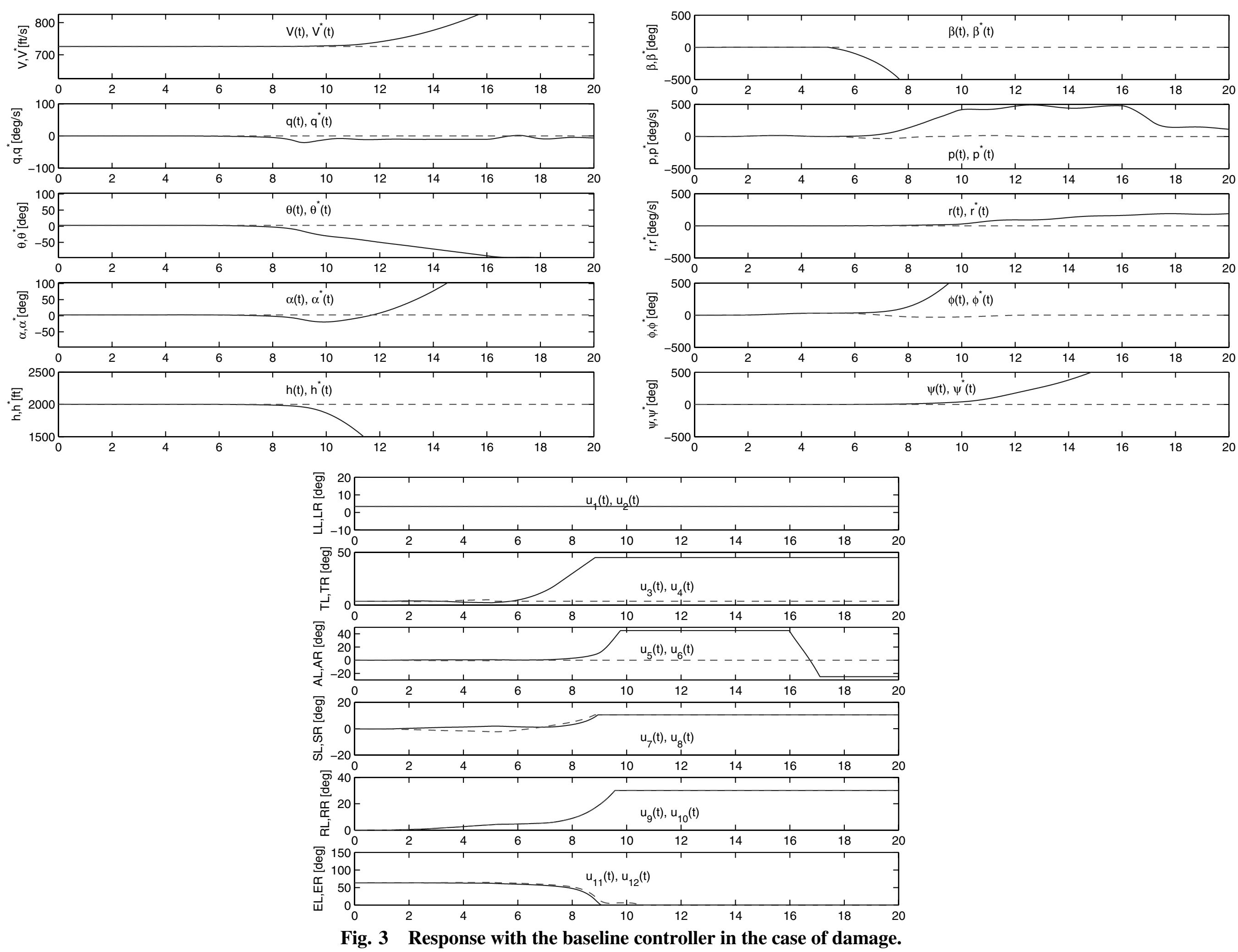

Fig. 3 Response with the baseline controller in the case of damage.

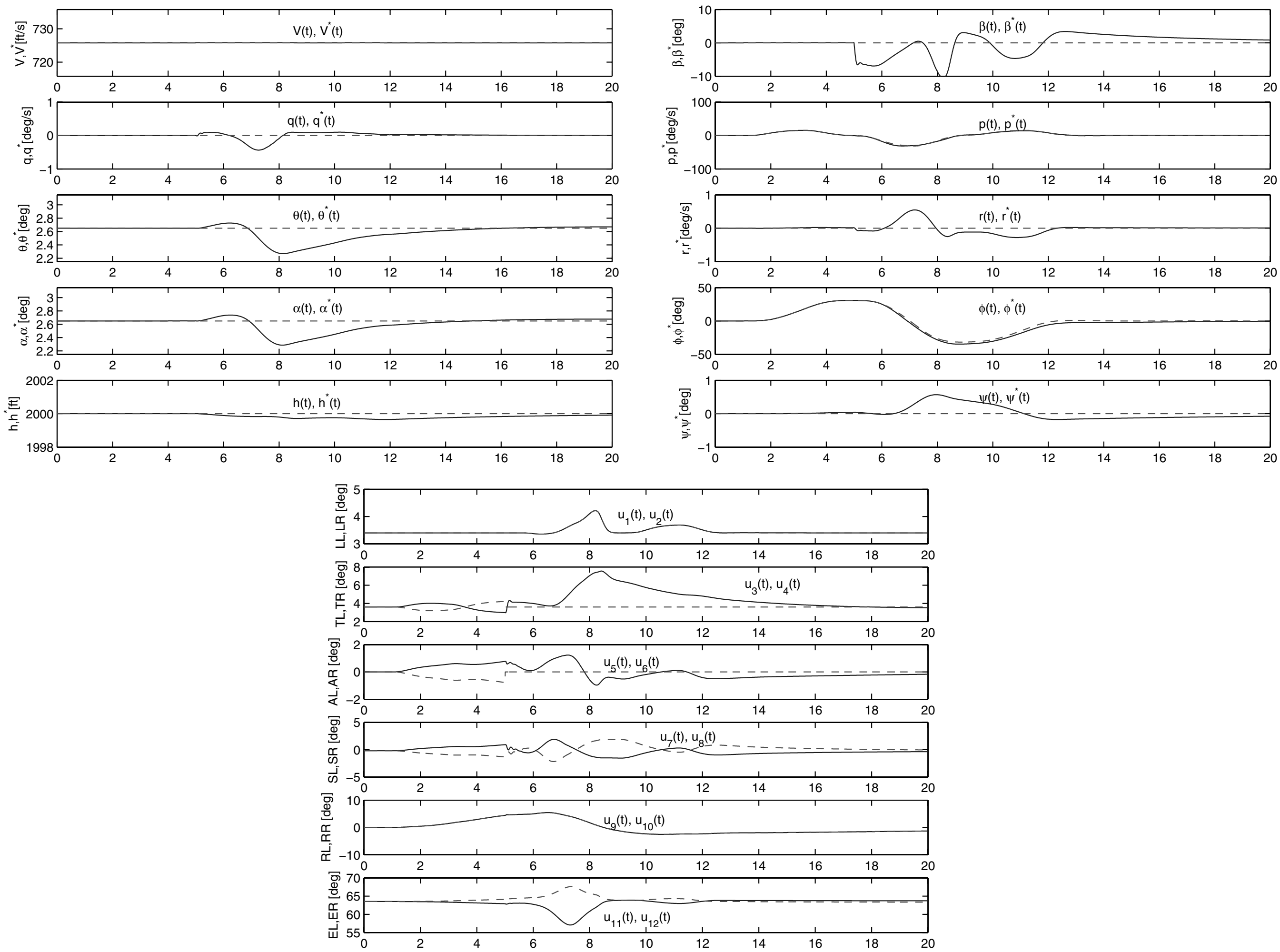

Fig. 4 Response with the retrofit fault-tolerant controller in the case of damage.

Damage Scenario: At $t = 5$ s into the maneuver, the right trailing-edge flap and right aileron undergo total damage (i.e., $d_4 = d_6 = 0$), and the following nonlinearity is generated due to the damage:

$$\xi = \begin{bmatrix} 4\xi_q & 4\xi_\eta & -6\xi_p & -4\xi_r & 0.1\xi_\beta \end{bmatrix}^T$$

where

$$\xi_q = x_5^2 + x_5 x_7 + x_5 x_8 + x_6^2 + x_{10}^2$$

$$\xi_\eta = 4x_6^2 + 0.5x_{10}^2$$

$$\xi_p = -6(x_7^2 + x_7 x_5 + x_7 x_8 + x_6^2 + x_{10}^2)$$

$$\xi_r = -4(x_8^2 + x_8 x_5 + x_7 x_8 + x_6^2 + x_{10}^2)$$

$$\xi_\beta = 0.1x_6^2 + 0.1x_{10}^2$$

This is a fairly severe nonlinearity that, if not compensated for, causes closed-loop system instability, as discussed later.

Some discussion regarding the previous nonlinearity is in order. Arriving at an accurate description of the postdamage aircraft dynamics is a truly formidable task as the effect of damage needs to be studied on an aeroservoelastic model coupled with rigid body dynamics. Our objective here is to demonstrate the potential of the proposed retrofit fault-tolerant control strategy to deal with the effect of damage, and not to address the modeling issue. Hence the above model may be an oversimplification of the true postdamage aircraft dynamics, but serves the purpose of demonstrating the feasibility of the approach and capturing some of the dominant effects of damage (e.g., additional rolling, yawing and pitching moments, and additional nonlinear effects on the angle of attack and slide-slip angle dynamics). The same discussion holds for $c_0(x)$ whose form can be deduced only after extensive wind-tunnel modeling under damage conditions.

Retrofit Fault-Tolerant Controller Implementation: To implement the controller, the Γ and Λ matrices are chosen as $\Gamma = 10I$ and $\Lambda = 4I$, while $c_0(x)$ is chosen as

$$c_0(x) = 45(|x_5| + |x_6| + |x_7| + |x_8| + |x_{10}|)^2$$

In addition, to avoid chattering the sign function is approximated as $\text{sign}(\zeta) \cong \zeta/(\|\zeta\| + 0.1)$.

Response with the Baseline Controller in the No-Damage Case: For this simulation, we chose an inverse dynamics model-reference controller as the baseline controller. Its objective is to track the output of a reference model that consists of four ssecond-order models with $\zeta = 0.7$ and $\omega = 2$ (for pitch, roll and yaw angles, and altitude), and two first-order models (for velocity and side-slip angle) whose steady-state gain is equal to 2. The response with the baseline controller is shown in Fig. 2. The actual aircraft response is shown as solid lines, while the desired response is shown as dashed lines. It is seen that the baseline controller achieves excellent response despite actuator dynamics and position and rate limits on the control effectors.

Response with the Baseline Controller in the Case of Damage: The above damage scenario was implemented in the case when the aircraft is controlled by the baseline controller. Because this controller does not have a mechanism to accommodate damage effects, the closed-loop system becomes unstable within a short time interval. In fact, the simulation breaks down after 2–3 s. Hence we included a simulation for a milder damage case, that is, a case when

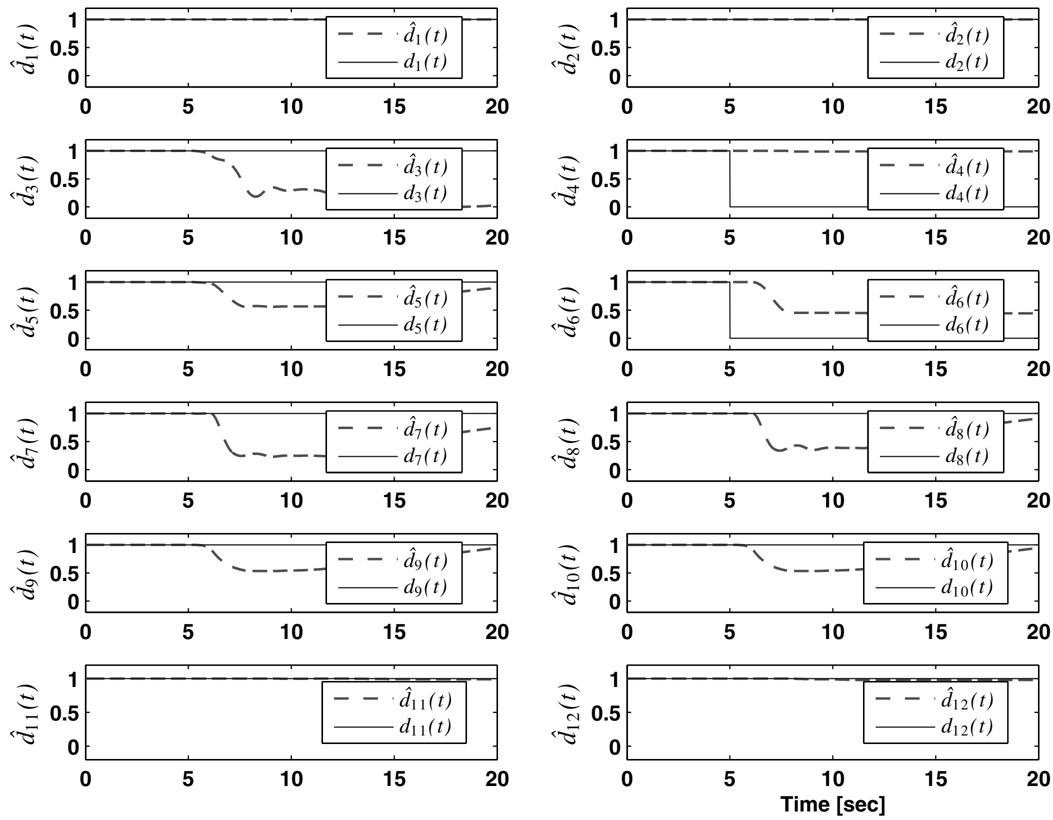

Fig. 5 Response of the damage parameter estimates.

there is no damage generated nonlinearity, but only the control effector damage. The resulting response is shown in Fig. 3, where it is seen that the system quickly becomes unstable because all the surfaces saturate. It can be concluded that a fault-tolerant control design is needed to compensate for the effect of damage.

Response with the Retrofit Fault-Tolerant Adaptive Controller: The state and control input response of the system in the damage case is shown in Fig. 4. It is seen that the controller effectively stabilizes the closed-loop system despite severe disturbances due to the damage and achieves acceptable performance. Figure 5 shows the

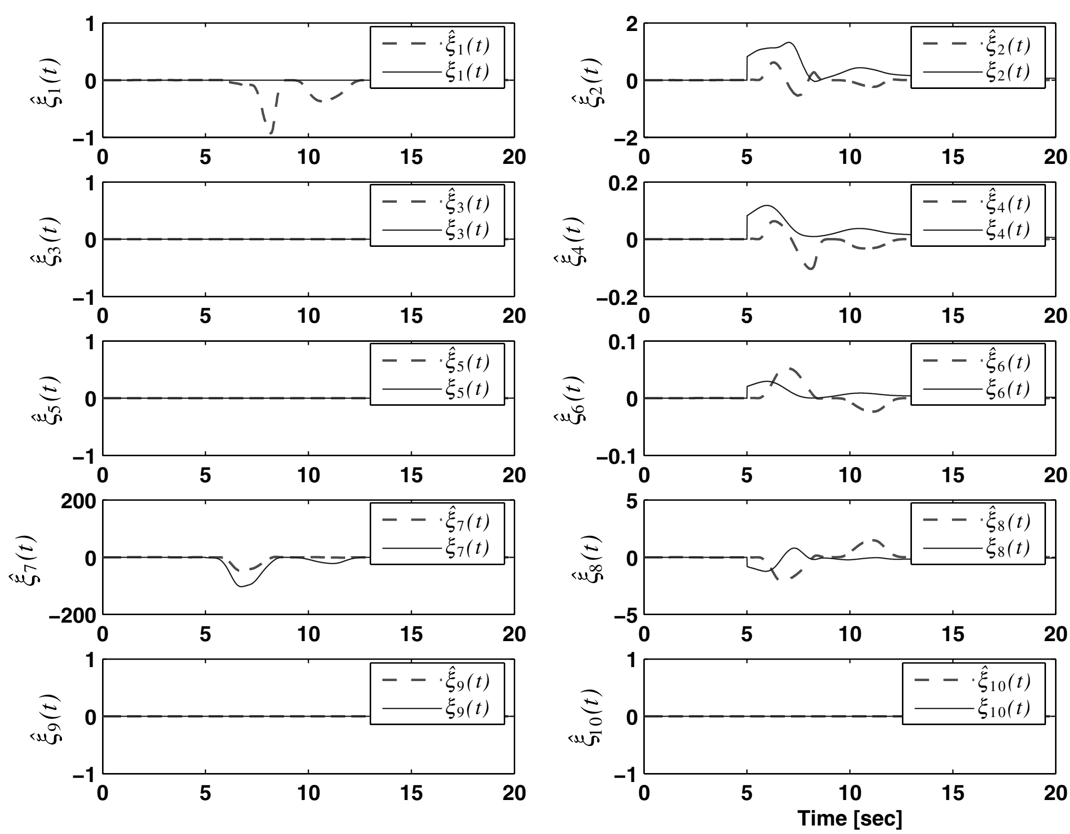

Fig. 6 Response of the variable-structure controller term.

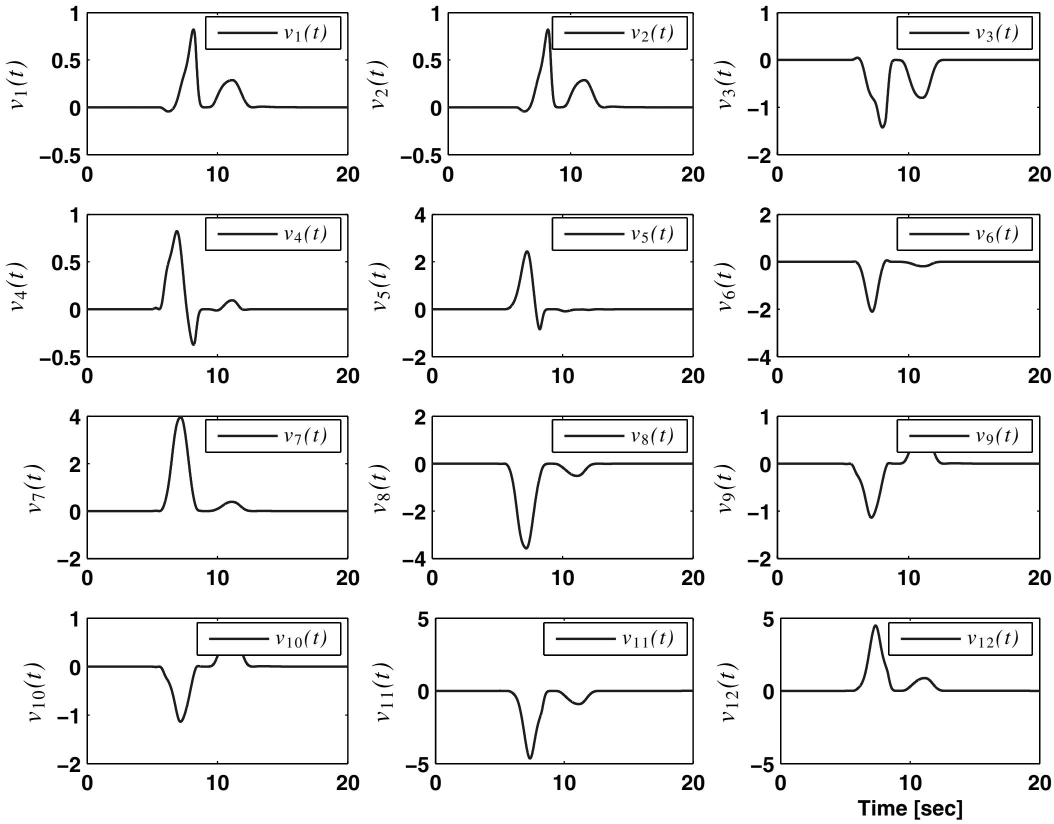

Fig. 7 Response of the retrofit signal.

response of the parameter estimates. It is seen that the estimates move only after the failure, which is a favorable feature of the proposed scheme. Also, it is seen that the actual damage is not estimated, that is, the estimates corresponding to damage parameters that have changed do not move, or move insufficiently, while the other estimates move. This is due to the following: 1) features of the proposed scheme where the damage is compensated by the collective effect of all parameter estimates, and 2) lack of the persistent excitation of the roll doublet in the 12-dimensional parametric space. As far as the first point is concerned, because an indirect adaptive control approach is used, persistent excitation is not needed to assure closed-loop stability and convergence of the tracking error to zero. The variable-structure term in the controller attempts to estimate the nonlinearity generated by the damage. This is seen in Fig. 6. It is this rapid compensation of large signals (note the size of the signals in the roll and yaw axes) generated by the damage that aids in the stabilization of the overall closed-loop control system. Figure 7 shows the response of the retrofit signal. It is seen that this signal is zero before the damage and becomes active only during damage accommodation. This is a desirable feature of the proposed controller.

V. Conclusions

In this paper a theoretical framework for retrofit reconfigurable flight control is developed and applied to a large class of nonlinear models of aircraft dynamics. The proposed approach solves a difficult problem of retrofit control design that retains a baseline controller and, at the same time, accommodates severe structural damage so that the overall system is stable and the control objective is met. This is accomplished using minimum prior information regarding the baseline controller. It is shown that the retrofit control design strongly depends on the properties of the nominal closed-loop system consisting of the unperturbed system dynamics and the dynamics of the nominal controller. Using those properties in a judicious manner, it is shown that a retrofit control signal that compensates for control effector damage and resulting state-dependent disturbances can be designed using the variable-structure adaptation of suitably chosen signals within the proposed retrofit control architecture. It is shown that under reasonable assumptions the retrofit control design results in zero tracking error in the presence of damage. A modification involving a convex optimization problem is presented for the case with control constraints.

The results from this paper have revealed that the proposed retrofit control design framework is suitable for reconfigurable flight control design. Initial analysis has also revealed that the proposed retrofit control architecture can be readily extended to the case of parametric uncertainty and higher-order unmodeled dynamics such as those arising in the context of flutter or flexible modes. Such extensions of the proposed algorithms will be the focus of future research in this area.

Appendix A: Proofs

This section gives the proof of the main results.

Proof of Theorem III.1: One first chooses a Lyapunov function candidate

$$V(\hat{e}_x, e_d) = \tfrac{1}{2}(\hat{e}_x^T \hat{e}_x + e_d^T \Gamma^{-1} e_d) \tag{A1}$$

Taking its first total derivative along the solutions of the system yields

$$\dot{V}(\hat{e}_x, e_d) \le -\hat{e}_x^T \Lambda \hat{e}_x \le 0$$

for all $(\hat{e}_x, e_d) \neq 0$, where we used the property of the adaptive algorithms with projection that $e_d^T \Gamma^{-1} \dot{e}_d \le -\hat{e}^T U e_d$ (see Appendix B). Hence $\hat{e}_x$ and $\hat{e}_d$ are bounded.

Denoting the minimum eigenvalue of Λ as λ, one further has

$$\dot{V}(\hat{e}_x, e_d) \le -\lambda \|\hat{e}_x\|^2$$

Upon integrating this equation from 0 to ∞, keeping in mind that $V(\infty)$ is bounded, one obtains that $\hat{e}_x \in \mathcal{L}^2$. □

Proof of Theorem III.2: From the definition of R from Eq. (39) and Assumptions 1.3 and 3 it follows that R is bounded for all values of the arguments. Using Assumption 3.1 it follows that z is bounded.

Hence $x_1 = z_1$ and $\omega = z_3$ are bounded. This also implies that x_2 is bounded (because $\hat{x}_2 = z_2$ and $\hat{e}_x = \hat{x}_2 - x_2$ are bounded). Because ω, x, and r are bounded, so is u_N. It can be readily verified that v is bounded as well, which implies that $u = u_N + v$ is bounded. Now from Eq. (27) it follows that $\dot{\hat{e}}_x$ is bounded. This, along with the result of Theorem 1 (i.e., that $\hat{e}_x \in \mathcal{L}^\infty \cap \mathcal{L}^2$) implies that $\hat{e}_x$ is uniformly continuous. Using Barbalat's Lemma ([10], p. 85) it follows that $\lim_{t\to\infty} \hat{e}_x(t) = 0$. Using Assumption 3.1 (ii) it follows that $\lim_{t\to\infty} \tilde{z}(t) = 0$. This implies that $\lim_{t\to\infty} e_x(t) = 0$ and $\lim_{t\to\infty} e_\omega(t) = 0$. □

Appendix B: Adaptive Algorithms with Projection

Properties of such adaptive algorithms will be illustrated on the example of a simple error model given as

$$\dot{e} = -\lambda e + \phi\omega(t, x)$$

where $\lambda > 0$, $e = x - x_m$, $x_m: \mathbb{R}^+ \to \mathbb{R}$ is a smooth bounded function, $\omega(t, x)$ is bounded for bounded x and for all time, $\phi = \theta - \theta^*$ denotes the parameter error, θ is an adjustable parameter, and $\theta^* \in [\theta_{\min}, \theta_{\max}]$ is constant. The objective is to adjust $\theta(t)$ within $[\theta_{\min}, \theta_{\max}]$ so that $\lim_{t\to\infty} e(t) = 0$.

Theorem A-1: If $\theta(t)$ is adjusted using the adaptive algorithm with projection of the form:

$$\dot{\theta} = \mathrm{Proj}_{[\theta_{\min},\theta_{\max}]}\{-\gamma e\omega\}, \qquad \theta(0) \in [\theta_{\min}, \theta_{\max}]$$

where the projection operator is defined as

$$\mathrm{Proj}_{[\theta_{\min},\theta_{\max}]}\{-\gamma e\omega\} = \begin{cases} -\gamma e\omega, & \text{if } \theta(t) = \theta_{\max} \text{ and } e\omega > 0, \\ 0, & \text{if } \theta(t) = \theta_{\max} \text{ and } e\omega \le 0, \\ -\gamma e\omega, & \text{if } \theta_{\min} < \theta(t) < \theta_{\max}, \\ 0, & \text{if } \theta(t) = \theta_{\min} \text{ and } e\omega \ge 0, \\ -\gamma e\omega, & \text{if } \theta(t) = \theta_{\min} \text{ and } e\omega < 0, \end{cases}$$

then $\lim_{t\to\infty} e(t) = 0$.

Proof: Let the tentative Lyapunov function for the system be

$$V(e, \phi) = \frac{1}{2}\left[e^2 + \frac{\phi^2}{\gamma}\right]$$

Its derivative along the motions of the system yields

$$\dot{V}(e, \phi) = -\lambda e^2 + e\phi\omega + \frac{\phi\dot{\phi}}{\gamma}$$

To assure that $\dot{V}$ is negative semidefinite, our objective is to show that in all cases

$$\phi\dot{\phi} \le -\gamma e\phi\omega$$

that is, $\phi\dot{\phi} + \gamma e\phi\omega \le 0$. We will further consider each individual case. We note that, because θ^* is constant, $\dot{\theta}(t) \equiv \dot{\phi}(t)$.

1) $\theta(t) = \theta_{\max}$. When $e\omega > 0$, we have that $\dot{\phi} = -\gamma e\omega$, and $\phi\dot{\phi} = -\gamma e\phi\omega$. Because $\phi = \theta - \theta^*$ and $\theta^* \in [\theta_{\min}, \theta_{\max}]$, in this case $\phi = \theta_{\max} - \theta^* \ge 0$. When $e\omega \le 0$, $\dot{\phi} = 0$, and we have $\phi\dot{\phi} + \gamma e\phi\omega = \gamma e\phi\omega \le 0$.

2) $\theta_{\min} < \theta(t) < \theta_{\max}$. In this case $\phi\dot{\phi} = -\gamma e\phi\omega$.

3) $\theta(t) = \theta_{\min}$. For $e\omega < 0$, we have that $\dot{\phi} = -\gamma e\omega$, and $\phi\dot{\phi} = -\gamma e\phi\omega$. Because in this case $\phi = \theta_{\min} - \theta^* \le 0$, for $e\omega \ge 0$, we have $\dot{\phi} = 0$, and $\phi\dot{\phi} + \gamma e\phi\omega = \gamma e\phi\omega \le 0$.

It follows that the adaptive algorithms with projection assure that the condition $\phi\dot{\phi} \le -\gamma e\phi\omega$ is satisfied for all values of arguments. This implies that $\dot{V}(e, \phi) \le -\lambda e^2 \le 0$. Using the standard arguments from, for example, [10], we can now readily demonstrate that $\lim_{t\to\infty} e(t) = 0$. □

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